Efficiency of On-the-Job Search in a Search and Matching Model with Endogenous Job Destruction *

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Abstract

I analyse the inefficiencies created in a search and matching model that allows for on-the-job search: First, the Hosios (1990) rule for the efficient level of the worker's bargaining power is adapted in a simple model. As the average gain of a new match is lower when some job seekers already have a job, the efficient level of labour market tightness should be lower and the worker's bargaining power higher than in a model devoid of on-the-job search. Second, the decision of when to perform on-the-job search is endogenised. It is shown that there is too much on-the-job search taking place because workers do not fully incorporate their current firms' loss when they quit. When partial wage commitment is introduced, the bargaining set becomes non-convex. Using a suitable bargaining process, I prove that wage commitment improves the efficiency of the on-the-job search decision and that the efficient level can be obtained.

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1 Introduction

This paper analyses the efficiency of on-the-job (OTJ) search in a search and matching model. OTJ search has been increasingly incorporated into search models to account for the large direct flows of workers from one job to another. For instance, Fallick and Fleischman (2004) find that nearly two fifths of new jobs in the U.S. are taken by previously employed workers.

I use a search model that allows for OTJ search and endogenous job destruction, which is close to the model outlined in Pissarides (2000). The main assumptions are that search is random and commitment in terms of wages and OTJ search is limited. Random search means that there is only one job market for firms and workers. Job seekers, however, differ in terms of their current employment status: unemployed workers benefit from finding a job more than employed job seekers, who are looking for a better job. If it were possible to separate these different groups of job seekers in a directed search model like Menzio and Shi (2011), then different submarkets would be created. It would be efficient to have relatively more vacancies for the unemployed to increase their chances to find a job. Without these separate markets, the efficient level of market tightness takes the potential gains of both the unemployed and the employed job-seekers into account. In section 2, the efficient level of market tightness is analysed in a simple model with OTJ search. In the spirit of the condition in Hosios (1990), I derive the optimal level of the worker's bargaining power. It is shown that, when OTJ search is present, it exceeds the level indicated by the Hosios rule. This is because the efficient level of market tightness is lower due to the trade-off described above that translates to the necessity of a higher worker's bargaining power.

The assumption of limited commitment creates another possible source of inefficiency: it is assumed that workers cannot commit to stay with their current firms but they can secretly perform OTJ search. Thereby, they impose an externality: the firm suffers a loss if its worker leaves for a better job. In contrast, a certain amount of OTJ search is efficient when the gain from a better job outweighs the loss of destroying the current job. This paper finds that in general there is too much OTJ search taking place. Only if firms can partially commit to the wage they pay, the efficient level can be reached. This is possible because by committing to a higher wage, the firm can make it less attractive for the worker to perform OTJ search. A result of this mechanism, as pointed out by Shimer (2006), is that the bargaining set becomes non-convex; when the firm pays a higher wage, the probability of the worker quitting is reduced, which increases the surplus of a match. A suitable bargaining game is solved to account for this non-convexity. The outcome reduces inefficient OTJ search and subsequently it is possible for the efficient level to be obtained.

Pissarides (1994) introduced OTJ search into a search model. My model differs from the one in Pissarides (2000) insofar as I do not allow for a joint OTJ search decision and I introduce partial commitment in terms of wages. The efficiency of OTJ search has been discussed in Stevens (2004). In her model, firms are allowed to post wage contracts that are more or less restricted. In the unrestricted case, workers pay a fee at the beginning of a match and then they get the whole rent. To allow for such contracts would yield an efficient OTJ search decision in my model, because employers do not care when a worker leaves. Hence, workers bear the full cost of destroying a match and choose the efficient threshold for performing OTJ search. Yet, Stevens (2004) concludes that turnover is too low, because workers do not incorporate the benefits a firm has from being matched. This is because she considers a partial equilibrium, in which firms and workers are matched at a constant rate. In general equilibrium with free entry of firms, however, these benefits are equal to the expected job creation cost and cancel out in an analysis of social welfare.

The result that there are too many vacancies is also obtained by Gautier et al. (2010), who consider a circular model of heterogeneity. When posting vacancies, firms do not take into account the loss of other firms whose employees are to be potentially hired.

In a different context, Moen and Rosén (2011) have adapted the Hosios rule, when workers have private information. A higher worker bargaining power of workers is desirable as it induces them to exert more effort. In this sense, they provide an alternative explanation for the result in proposition 1.

There are two approaches for wage bargaining in models including OTJ search: Cahuc et al. (2006) allow for counter-offers when employed workers get another job offer. Workers can then benefit from the competition between the two possible employers. In this paper, I follow Shimer (2006), for instance, and exclude such competition.

Another approach for modelling OTJ search is the class of directed search models like in Menzio and Shi (2011) and Menzio and Shi (2010). The main advantage of the so called block recursive equilibrium they obtain, is the tractability of the solution. Introducing different sub-markets for job seekers then yields an efficient level of market tightness for each group.

The paper is structured as follows: section 2 discusses a simple version of the model that abstracts from the decisions regarding OTJ search and job destruction. The adaptation of the Hosios rule needed to obtain the efficient level of market tightness is derived. The full model with endogenous job destruction and OTJ search decision is analysed in section 3. In Section 4, partial commitment is introduced, and the bargaining process is adapted to account for the non-convex bargaining set. Finally, section 5 concludes.

2 Simple Model

2.1 Setup

The simple model I consider first is a special case of the endogenous job destruction model in Pissarides (2000) that allows for OTJ search. It is a continuous time search and matching model. The mass of workers is normalised to 1. Firms and workers are risk-neutral, having a discount rate r. Every firm-worker pair can have two possible levels of productivity: all jobs are created at productivity p. At rate λ, shocks arrive that reduce the productivity to px. Jobs of either productivity are destroyed at the exogenous rate δ.

Vacancies and jobseekers are matched according to a constant returns to scale matching function $m(u+e, v)$, where unemployed and employed job seekers are equally likely to be matched.

From the matching function it follows that job seekers find a new job at rate $a(\theta) = m(1, \theta)$, where $\theta \equiv \frac{v}{u+e}$ denotes labour market tightness. This is the ratio of vacancies to the total number of job seekers, who consist of the unemployed u and the OTJ seekers e. The function $a(\theta)$ is increasing and concave in θ and the Inada conditions shall hold.[1] The arrival rate of employees to a vacancy is then given by the rate $q(\theta) = \frac{a(\theta)}{\theta}$.

The flow cost of maintaining a vacancy for a firm is pc. Unemployed workers receive a flow payoff of b. When employed workers want to perform OTJ search, they incur the cost ν.[2]

The purpose of this simple model is to demonstrate how the efficient level of market tightness is influenced by the existence of OTJ search. I derive the generalisation of the Hosios (1990) rule for the worker's bargaining power that ensures an efficient market outcome. When there is OTJ search, the additional surplus of finding a new job for employed workers is lower than the surplus for the unemployed. Therefore, OTJ seekers are relatively more patient to find a better job. In a directed search model like Menzio and Shi (2011) this implies that in equilibrium the market for OTJ seekers features a lower market tightness (i.e. a lower job-finding rate), but a higher wage. As I do not allow for directed search, it is intuitive that the optimal market tightness will lie between the low level of market tightness for OTJ seekers and the high level for the unemployed in a directed search model. To get an efficient market outcome, the worker's bargaining power will then have to be higher than the one suggested by the Hosios (1990) rule. Otherwise, market tightness in the decentralised economy is too high as an OTJ seeker does not fully incorporate the negative impact a change of jobs has on her previous employer.

As I am interested in this efficient level of market tightness, I want to eliminate other decisions made by the worker and the firm, notably when to perform OTJ search and when to destroy a job. Therefore, I assume that the parameters are such that it is both socially and individually optimal to perform OTJ search and not to destroy the job after a λ-shock has decreased the productivity of a match.[3] In the full model from Section 3 onwards, this assumption is relaxed and performing OTJ search becomes an endogenous choice.

First, the efficient solution given by the social planner's problem is derived, then the market outcome is derived and the two are compared.

2.2 Social Planner's problem

A social planner solves the following optimal control problem: the state variables are the fraction of unemployed, employed at productivity p, and employed at productivity px denoted by u, w_1, and w_x, respectively ($u + w_1 + w_x = 1$). The only control variable in the simple model is labour market tightness θ. Given the initial distribution across states, she wants to maximise the present discounted value of the flow benefits for the unemployed plus output produced by

[1] That is $\lim_{\theta \to 0} a'(\theta) = \infty$ and $\lim_{\theta \to \infty} a'(\theta) = 0$.

[2] In this section I impose that OTJ search always takes place at the lower level of productivity. Without loss of generality, OTJ search costs could therefore be set to 0 and be deducted from the output, px. I include them to make the equations better comparable to those in the following sections.

[3] The latter condition is simply $b \leq px - \nu$ as the low productivity is an absorbing state.

employed less the cost of opening vacancies and of OTJ search:

$$\max \int_0^\infty e^{-rt} \left[ub + p\left(w_1 + x w_x\right) - pc\theta\left(u + w_x\right) - \nu w_x\right] dt, \tag{1a}$$

$$\dot{u} = \delta\left(w_1 + w_x\right) - a\left(\theta\right) u, \tag{1b}$$

$$\dot{w}_1 = -\left(\delta + \lambda\right) w_1 + a\left(\theta\right)\left(u + w_x\right), \tag{1c}$$

$$\dot{w}_x = -\left(\delta + a\left(\theta\right)\right) w_x + \lambda w_1. \tag{1d}$$

The stock of unemployed workers increases, because employed workers lose their job at rate δ, and it decreases, because unemployed find a job at rate $a(\theta)$. The inflow into employment at productivity 1 is given by the fraction of unemployed and OTJ seekers who have found a new job, whereas a fraction $\delta + \lambda$ lose their job dues to the shocks. Finally, the inflow into low productivity jobs is only due to λ-shocks whereas these jobs are lost at a rate $\delta + a(\theta)$ due to job destruction and OTJ search, respectively.

Denote by U, W_1, and W_x the respective costates for the three state variables. The Hamiltonian in the current-value form is given by

$$\begin{aligned} H = \; & ub + p\left(w_1 + x w_x\right) - pc\theta\left(u + w_x\right) - \nu w_x + U\left[\delta\left(w_1 + w_x\right) - a\left(\theta\right) u\right] \\ & + W_1\left[-\left(\delta + \lambda\right) w_1 + a\left(\theta\right)\left(u + w_x\right)\right] + W_x\left[-\left(\delta + a\left(\theta\right)\right) w_x + \lambda w_1\right]. \end{aligned} \tag{2}$$

The first order condition for the optimal choice of market tightness is given by

$$a'\left(\theta\right)\left[u\left(W_1 - U\right) + w_x\left(W_1 - W_x\right)\right] = pc\left(u + w_x\right), \tag{3a}$$

$$\left(1 - \eta\right)\left[\frac{u}{u + w_x}\left(W_1 - U\right) + \frac{w_x}{u + w_x}\left(W_1 - W_x\right)\right] = \frac{pc}{q\left(\theta\right)}, \tag{3b}$$

where $\eta \equiv 1 - \frac{a'(\theta)\theta}{a(\theta)}$ is equal to the elasticity of the matching function with respect to the number of job seekers. The left hand side of the first equation describes the marginal benefit of a higher job arrival rate by increasing θ. In optimum it must be equal to the marginal cost on the right hand side. In the rearranged second line, one can see that the actual benefit of higher market tightness is given by the weighted average of the gains for an unemployed and for an employed job seeker, as there is only one market at which job seekers can be matched.

The costates' differential equations are given by

$$\dot{U} = rU - \left[b - pc\theta + a\left(\theta\right)\left(W_1 - U\right)\right], \tag{4a}$$

$$\dot{W}_1 = rW_1 - \left[p + \delta\left(U - W_1\right) + \lambda\left(W_x - W_1\right)\right], \tag{4b}$$

$$\dot{W}_x = rW_x - \left[px - pc\theta - \nu + a\left(\theta\right)\left(W_1 - W_x\right) + \delta\left(U - W_x\right)\right]. \tag{4c}$$

I focus on the description of a steady state, in which the states and costates are constant. Then the equations reduce to:

$$rU = b - pc\theta + a(\theta)(W_1 - U), \tag{5a}$$

$$rW_1 = p + \delta(U - W_1) + \lambda(W_x - W_1), \tag{5b}$$

$$rW_x = px - pc\theta - \nu + a(\theta)(W_1 - W_x) + \delta(U - W_x). \tag{5c}$$

The first equation states that the flow value of an unemployed worker is given by the unemployment benefit less the proportional cost of maintaining vacancies plus the option value of getting matched at rate $a(\theta)$. A match at the high level of productivity benefits from output p but at rate δ the match is destroyed and at rate λ the productivity is downgraded. For a low productivity match, the output px has to be reduced by the direct OTJ search costs ν and the cost for the vacancies. A better job is found at rate $a(\theta)$ and at rate δ the job breaks down. One can rearrange these value equations to obtain the respective surplus:

$$(r + a(\theta) + \delta)(W_x - U) = px - b - \nu, \tag{6a}$$

$$(r + a(\theta) + \lambda + \delta)(W_1 - W_x) = p(1 - x) + pc\theta + \nu, \tag{6b}$$

$$(r + a(\theta) + \delta)(W_1 - U) = p - b + pc\theta + \lambda(W_x - W_1). \tag{6c}$$

From the laws of motion for the states, one can obtain the distribution across states in a steady state:

$$u = \frac{\delta}{\delta + a(\theta)}, \tag{7a}$$

$$w_1 = \frac{a(\theta)}{\delta + \lambda + a(\theta)}, \tag{7b}$$

$$w_x = \frac{\lambda}{\delta + a(\theta)} w_1. \tag{7c}$$

Higher level of market tightness means that job seekers are matched faster, reducing the steady state value of unemployed and of low quality matches relative to high quality matches. The level of high quality matches then increases. Substituting the steady state value of the states and the values obtained from equations (6b) and (6c) into the first order condition (3b) for θ, implicitly yields the efficient steady state value for market tightness.

2.3 Decentralised economy

This subsection determines the equilibrium in the decentralised economy. The value of an unemployed and an employed worker at the two levels of productivity is denoted by U, W_1, and

W_x, respectively. Her wage is denoted by $\bar{w}_1$ and $\bar{w}_x$. This gives the stationary value equations

$$\begin{aligned} rU &= b + a(\theta)(W_1 - U), && \text{(8a)} \\ rW_1 &= \bar{w}_1 + \delta(U - W_1) + \lambda(W_x - W_1), && \text{(8b)} \\ rW_x &= \bar{w}_x - \nu + a(\theta)(W_1 - W_x) + \delta(U - W_x). && \text{(8c)} \end{aligned}$$

They differ from the social planner's equations (5a), (5b), and (5c) only insofar as job seekers do not directly bear the cost of vacancies and employed workers receive a wage instead of the whole output. These can be rewritten in terms of the surplus:

$$\begin{aligned} (r + a(\theta) + \delta)(W_1 - U) &= \bar{w}_1 - b + \lambda(W_x - W_1), && \text{(9a)} \\ (r + a(\theta) + \lambda + \delta)(W_1 - W_x) &= \bar{w}_1 - \bar{w}_x + \nu. && \text{(9b)} \end{aligned}$$

Firms can open and maintain vacancies at cost pc, getting matched at rate $q(\theta)$. The value of a vacancy is denoted by V. The firm's value of a match is denoted by J_1 and J_x, respectively. Its flow profits are given by output less wage. A match with high productivity is destroyed at rate δ and downgraded at rate λ. A low productivity match is destroyed exogenously at rate δ and destroyed endogenously due to the worker leaving for another firm at rate $a(\theta)$. This yields the following value equations:

$$\begin{aligned} rV &= -pc + q(\theta)(J_1 - V), && \text{(10a)} \\ rJ_1 &= p - \bar{w}_1 - \delta J_1 + \lambda(J_x - J_1), && \text{(10b)} \\ rJ_x &= px - \bar{w}_x - (\delta + a(\theta)) J_x. && \text{(10c)} \end{aligned}$$

The last equation can be solved for the value of low productivity match:

$$J_x = \frac{px - \bar{w}_x}{r + \delta + a(\theta)}. \tag{11}$$

In equilibrium, rents from opening vacancies are exhausted such that the zero profit condition holds:

$$J_1 = \frac{pc}{q(\theta)}. \tag{12}$$

Using this condition, one can obtain the analogon of equation (9b) for the firm:

$$(r + a(\theta) + \lambda + \delta)(J_1 - J_x) = p(1 - x) - (\bar{w}_1 - \bar{w}_x) + pc\theta. \tag{13}$$

The equilibrium wage is determined by Nash bargaining. The worker gets a share β, with the remainder going to the firm. It is assumed that the wage can be continuously renegotiated. Most importantly, it is adjusted after a negative productivity shock and it is not possible for the firm to prevent OTJ search by committing to a higher wage. From the sharing rule one obtains the respective conditions at the two types of matches:

$$(1-\beta)(W_1-U) = \beta J_1, \tag{14a}$$
$$(1-\beta)(W_x-U) = \beta J_x. \tag{14b}$$

Taking the difference of the two and substituting from equations (9b) and (13), one obtains the equation for the wage differential:

$$(1-\beta)(W_1-W_x) = \beta(J_1-J_x), \tag{15a}$$
$$(1-\beta)(\bar{w}_1-\bar{w}_x+\nu) = \beta[p(1-x)-(\bar{w}_1-\bar{w}_x)+pc\theta], \tag{15b}$$
$$\bar{w}_1-\bar{w}_x = \beta[p(1-x)+pc\theta]-(1-\beta)\nu. \tag{15c}$$

Similarly, using equations (9a) and (10b), the wage at the high productivity is derived:

$$(1-\beta)[\bar{w}_1-b+\lambda(W_x-W_1)] = \beta[p-\bar{w}_1+a(\theta)J_1+\lambda(J_x-J_1)], \tag{16a}$$
$$(1-\beta)(\bar{w}_1-b) = \beta(p-\bar{w}_1+pc\theta), \tag{16b}$$
$$\bar{w}_1 = (1-\beta)b+\beta(p+pc\theta). \tag{16c}$$

This is the standard wage equation, which is not affected by the introduction of productivity shocks and OTJ search, as it is renegotiated after shocks. Substituting it into the equation for the wage differential gives the wage at the low level of productivity:

$$\bar{w}_x = (1-\beta)(b+\nu)+\beta px. \tag{17}$$

The worker is compensated for foregoing unemployment benefits and incurring OTJ search cost ν. In addition, she gets a share of the output.

The job creation condition is obtained by equating the value J_1 from equation (10b) in combination with equation (11) to the value from the zero profit condition (12):

$$p-\bar{w}_1+\lambda\frac{px-\bar{w}_x}{r+\delta+a(\theta)} = (r+\delta+\lambda)\frac{pc}{q(\theta)}. \tag{18}$$

Substituting the wage equations derived above into this equation, yields the equilibrium level of market tightness. Given market tightness, the steady state distribution across employment states is given by the same equations (7a), (7b), and (7c) as in the social planner's case. As labour market tightness is the only control variable in this model, the difference between the efficient outcome and the decentralised equilibrium can be analysed by comparing θ in the steady state. This is done in the next subsection.

2.4 Efficiency condition

Along the lines of Hosios (1990), I want to find the condition for the bargaining power that must hold in order to make the decentralised equilibrium efficient. This means that $\theta^{SP}=\theta^{DEC}$ shall hold where superscript SP denotes variables in the social planner's outcome and DEC denotes variables in the decentralised equilibrium. The social planner's first order condition (3b) is

$$(1-\eta)\left[\frac{u}{u+w_x}\left(W_1^{SP}-U^{SP}\right)+\frac{w_x}{u+w_x}\left(W_1^{SP}-W_x^{SP}\right)\right]=\frac{pc}{q\left(\theta^{SP}\right)}. \tag{19}$$

In the decentralised economy, the zero profit condition (12) and the Nash sharing rule imply

$$\frac{pc}{q\left(\theta^{DEC}\right)}=(1-\beta)\left(W_1^{DEC}+J_1^{DEC}-U^{DEC}\right). \tag{20}$$

Note that when market tightness is the same in both cases the surplus of a job is also the same:

$$W_1^{SP}\left(\theta^{SP}\right)-U^{SP}\left(\theta^{SP}\right)=W_1^{DEC}\left(\theta^{SP}\right)+J_1^{DEC}\left(\theta^{SP}\right)-U^{DEC}\left(\theta^{SP}\right). \tag{21}$$

Combining the three equation, yields the condition for efficiency:

$$(1-\eta)\left[\frac{u}{u+w_x}\left(W_1^{SP}-U^{SP}\right)+\frac{w_x}{u+w_x}\left(W_1^{SP}-W_x^{SP}\right)\right]=(1-\beta)\left(W_1^{SP}-U^{SP}\right). \tag{22}$$

This can be rearranged to express the efficient level of bargaining power:

$$\beta=\eta+(1-\eta)\frac{w_x}{u+w_x}\frac{W_x^{SP}-U^{SP}}{W_1^{SP}-U^{SP}}. \tag{23}$$

The Hosios condition without OTJ search is $\beta=\eta$. This is too low when OTJ search is present (i.e. $w_x>0$) and the value of a low productivity match is bigger than the value of an unemployed (i.e. $W_x^{SP}>U^{SP}$). The latter condition is equivalent to the gain of matched OTJ seekers being lower than the one of matched unemployed workers. The higher this deviation is and the more relevant it is due to more OTJ seekers, the larger is the optimal bargaining power of the worker. The intuition behind this result is that a high bargaining power increases wages thereby reducing the profits of firms and consequently the number of vacancies they open. Otherwise there would be too many vacancies; firms do not account for the fact that when opening a vacancy the rate at which jobs are destroyed endogenously is increased, thereby imposing a negative externality on existing firms. This additional externality is corrected for by adapting the Hosios rule.

The result above proves the following proposition:

Proposition 1 *In an economy with on-the-job search, the efficient level of the worker's bargaining power is larger than the elasticity of the matching function with respect to the number of job seekers:*

$$\beta=\eta+(1-\eta)\frac{w_x}{u+w_x}\frac{W_x^{SP}-U^{SP}}{W_1^{SP}-U^{SP}}$$

In this section, I implicitly restricted the parameters such that OTJ search is efficient. In the next section, I discuss the general model, in which both the decision to destroy jobs and the decision to perform OTJ search are determined endogenously.

3 Model with endogenous job destruction and on-the-job search

I now relax the assumption that there are only two possible values for the productivity of a match. The model is similar to the one in chapter 4 of Pissarides (2000). Job destruction is endogenised along the lines of Mortensen and Pissarides (1994), which also makes the OTJ search decision endogenous. I assume that there is a finite number of productivity levels px_i ($x_1 = 1 > x_2 > \ldots > x_n$) and jobs are still created at the maximum productivity p. Shocks that arrive at rate λ change the productivity of a match where the new productivity is drawn according to the probability mass function $g(x_i)$ ($\sum_{i=1}^{n} g(x_i) = 1$).[4] Jobs are destroyed endogenously if the new productivity level is below a certain reservation productivity such that all jobs with a lower productivity are destroyed. At each level of productivity, a worker can decide whether she wants to perform OTJ search. This will determine a second threshold such that OTJ search takes place for all matches with a lower productivity.

I start again with the discussion of the efficient solution and then I find the market outcome and compare the two.

3.1 Social Planner's problem

Compared to the simple model in section 2.2, the planner has additional controls in the full model. For each level of productivity, let the dummy variable e_i denote the OTJ search decision such that OTJ search takes place when $e_i = 1$. Similarly, let the dummy variable d_i denote the destruction decision for matches with productivity x_i.[5] The maximisation problem takes the following form:

$$\max \int_0^\infty e^{-rt} \left[ub + p \sum_{i=1}^{n} x_i w_i - pc\theta \left(u + \sum_{i=1}^{n} e_i w_i \right) - \nu \sum_{i=1}^{n} e_i w_i \right] dt, \tag{24a}$$

$$\dot{u} = \lambda \sum_{i=1}^{n} d_i g(x_i) \sum_{j=1}^{n} w_j - a(\theta) u, \tag{24b}$$

$$\dot{w}_1 = \lambda \left(g(x_1) \sum_{j=1}^{n} w_j - w_1 \right) + a(\theta) \left(u + \sum_{i=1}^{n} e_i w_i \right), \tag{24c}$$

$$\dot{w}_i = \lambda \left((1 - d_i) g(x_i) \sum_{j=1}^{n} w_j - w_i \right) - e_i a(\theta) w_i. \tag{24d}$$

As above, the flow utility is given by unemployment benefits and output less the costs for opening vacancies and OTJ search costs.[6] Similar to the simple model, the inflow into unemployment is

[4] Note that the simple model above is nested in this more general model by setting $x_2 = x$, $x_3 = 0$, $g(x_2) = \frac{\lambda}{\lambda+\delta}$, $g(x_3) = \frac{\delta}{\lambda+\delta}$, and $\tilde{\lambda} = \lambda + \delta$.

[5] The linearity of the objective function and the constraints in e_i and d_i ensures that a corner solution is optimal.

[6] The laws of motion are written for stationary d_i. If for example an exogenous productivity shock raised the job destruction threshold, there would be a mass of jobs destroyed, which is not incorporated in the differential equations.

given by workers that lose their jobs after a productivity shock, and the outflow is given by the unemployed that are newly matched. The stock of workers at each level of productivity changes, because after a λ-shock workers' productivity gets changed. Additionally, there is an outflow if OTJ search takes place and at the highest productivity there is an inflow due to newly matched unemployed and OTJ seekers.

The value of an unemployed is still denoted by U, and W_i denotes the value of a match with productivity x_i. Appendix A.1 derives the Bellman equations, which are analogous to equations (5a), (5b), and (5c) in the simple model:

$$\begin{aligned} rU &= b - pc\theta + a(\theta)(W_1 - U), && (25a)\\ rW_1 &= p + \lambda\left(\sum_{j=1}^{n} g(x_j)\left[d_jU + (1-d_j)W_j\right] - W_1\right), && (25b)\\ rW_i &= px_i + e_i\left[a(\theta)(W_1 - W_i) - pc\theta - \nu\right] \\ &\quad +\lambda\left(\sum_{j=1}^{n} g(x_j)\left[d_jU + (1-d_j)W_j\right] - W_i\right). && (25c)\end{aligned}$$

The main differences compared to the value equations obtained in the simple model are that the option value of OTJ search is only present when $e_i = 1$ and that there is a new option value after a λ-shock: the latter is the expectation over getting the value W_j of a match at a new productivity level or the value U of an unemployed worker, if the job is optimally destroyed in response to a λ shock.

Appendix A.1 derives the threshold $\bar{S}(\theta)$, such that OTJ search is optimal for all matches with $x \le \bar{S}(\theta)$:

$$\bar{S}(\theta) = 1 - \frac{(pc\theta + \nu)(r + \lambda)}{a(\theta)p}. \quad (26)$$

Without fixed OTJ search costs ν, the threshold would be decreasing in θ because of the decreasing marginal returns of the matching function.[7] The intuition behind this negative relationship is that the expected cost of getting a new job $(\frac{pc}{q(\theta)})$ is increasing in θ. This makes OTJ search profitable for fewer types of matches. The threshold for OTJ search determines the controls e_i given θ:

$$e_i = \begin{cases} 1 & \text{if } x_i \le \bar{S}(\theta) \\ 0 & \text{if } x_i > \bar{S}(\theta) \end{cases}. \quad (27)$$

Now the threshold for job destruction can be determined. There exists such a threshold, because matches are identical except for their productivity. Therefore, if matches at some productivity level are destroyed, also matches at lower levels of productivity have to be destroyed in the optimum. I assume that the parameters are such that there is at least some efficient OTJ search at the worst non-destroyed matches.[8] Then it follows from equations (81a) and (81c),

[7] In general $sgn[S'(\theta)] = -sgn[\eta pc\theta - \nu(1-\eta)]$ so that it is decreasing for θ large enough. Otherwise, the fixed search cost is too high given a low probability of getting matched.

[8] If OTJ search costs were prohibitively high, the outcome would be identical to the model without OTJ search.

that matches are destroyed if

$$\frac{p\left(1-x_i\right)+\nu+pc\theta}{r+a\left(\theta\right)+\lambda}\geq W_1-U. \tag{28}$$

The left hand side is the difference between W_1 and W_i which is determined by the actual difference in productivity and the cost of vacancies and OTJ search multiplied by the average (discounted) time until OTJ search is successful or a productivity shock has arrived. If this difference becomes larger than the surplus of a new match, the job is destroyed. Note that $(W_1 - U)$ itself depends on the threshold. Inequality (28) thus implicitly determines the threshold $R(\theta)$, which yields the controls:

$$d_i=\begin{cases}1 & \text{if } x_i\leq R\left(\theta\right)\\ 0 & \text{if } x_i>R\left(\theta\right)\end{cases}. \tag{29}$$

The steady state distribution across employment states can be obtained analogously to the simple model, yielding:

$$u = \frac{\delta}{\delta+a\left(\theta\right)}, \tag{30a}$$

$$w_1 = g\left(x_1\right)\frac{a\left(\theta\right)}{\delta+a\left(\theta\right)}+\frac{a\left(\theta\right)\left(u+e\right)}{\lambda}, \tag{30b}$$

$$w_i = \begin{cases}\frac{\lambda}{\lambda+e_i a(\theta)}g\left(x_i\right)\frac{a(\theta)}{\delta+a(\theta)} & \text{if } d_i=0\\ 0 & \text{if } d_i=1\end{cases}. \tag{30c}$$

For a given value θ, the optimal values for d_i and e_i, the values for the surplus W_1-U and W_1-W_i, as well as the distribution across states u and w_i have thus been determined. Using this for the first order condition (76b), one can find the possible steady states for labour market tightness θ. Depending on the parameters, there is not necessarily a unique steady state: intuitively, multiple steady states can occur, because higher labour market tightness usually lowers the OTJ search threshold. This could increase the average gain from matching the average job seeker as workers with higher productivity do not perform OTJ search anymore. The higher cost arising from more vacancies per job seeker could thus be compensated by a higher average gain. In the steady state with low market tightness, there would be relatively more unemployed workers but more employed workers at a high level of productivity as more workers perform OTJ search. If there is not a unique steady state, the optimal choice might even depend on the initial distribution across employment states. It could be interesting to find and analyse the outcome in such cases, which is beyond the scope of this paper.

3.2 Decentralised equilibrium without commitment

In this subsection, the market outcome is determined when there is no commitment in terms of wages or the OTJ search decision. Workers determine privately if they want to perform OTJ search. As the wage is continuously renegotiated, they base their decisions on the market wage that is prevailing at the different levels of productivity.[9] In this case, a higher wage cannot prevent the worker from OTJ search. This results in too much OTJ search in equilibrium as the worker does not take fully into account the loss that the firm incurs when she leaves. In contrast, the decision for job destruction is still constrained efficient, because wages are determined by Nash bargaining. Then the firm's or the worker's surplus is negative if and only if the combined surplus is negative.

The stationary value equations for the worker are now given by

$$rU = b + a(\theta)(W_1 - U), \tag{31a}$$

$$rW_1 = \bar{w}_1 + \lambda \sum_{j=1}^{n} g(x_j)(\max\{W_j, U\} - W_1), \tag{31b}$$

$$rW_i = \bar{w}_i + \lambda \sum_{j=1}^{n} g(x_j)(\max\{W_j, U\} - W_i) + \max\{a(\theta)(W_1 - W_i) - \nu, 0\}. \tag{31c}$$

They differ from the simple model's equations insofar as the worker can choose whether or not to perform OTJ search, and the value after a λ-shock is given by the expectation over the outcome in all possible states. The equations can be rearranged to obtain expressions for the surplus:

$$(r + \lambda + e_i a(\theta))(W_1 - W_i) = \bar{w}_1 - \bar{w}_i + e_i \nu, \tag{32a}$$

$$(r + \lambda + a(\theta))(W_1 - U) = \bar{w}_1 - b + \lambda \sum_{j=1}^{n} g(x_j)\max\{W_j - U, 0\}. \tag{32b}$$

The firms' equation for the value of a vacancy does not change. The equations for the firms' value of a match have to be similarly adjusted to account for the λ-shock. Additionally, a job can be destroyed at rate $a(\theta)$ if the worker performs OTJ search (i.e. $e_i = 1$):

$$rV = -pc + q(\theta)(J_1 - V), \tag{33a}$$

$$rJ_1 = p - \bar{w}_1 + \lambda \sum_{j=1}^{n} g(x_j)(\max\{J_j, 0\} - J_1), \tag{33b}$$

$$rJ_i = px_i - \bar{w}_i + \lambda \sum_{j=1}^{n} g(x_j)(\max\{J_j, 0\} - J_i) - e_i a(\theta) J_i. \tag{33c}$$

The first equation implies the same zero profit condition as in the simple problem:

$$J_1 = \frac{pc}{q(\theta)}. \tag{34}$$

Making use of it, the firms' surplus of a high productivity match over a lower productivity match can be calculated in equilibrium:

$$(r + \lambda + e_i a(\theta))(J_1 - J_i) = p(1 - x_i) - (\bar{w}_1 - \bar{w}_i) + e_i pc\theta. \tag{35}$$

[9] Nevertheless, the privately optimal decision of the worker is anticipated in the bargaining problem, and the surplus is calculated based on it.

The wage is again determined by Nash-bargaining with the worker's share being β. The possible problem of a non-convex bargaining set as pointed out in Shimer (2006) does not arise here: the continuation value and the OTJ search decision only depend on the equilibrium values. Therefore, the wage bargaining only splits the output less possible OTJ search costs in the current (infinitesimal) period.[10] As the combined surplus is not influenced by the outcome of the bargaining process, the bargaining set is convex. In the next section, I relax the assumption of no commitment leading to a possible non-convexity.

The analogon of the wage differential (15c) can be derived from equations (32a) and (35) using the Nash sharing rule:

$$\bar{w}_1 - \bar{w}_i = \beta \left[p \left(1 - x_i\right) + e_i p c \theta\right] - \left(1 - \beta\right) e_i \nu. \tag{36}$$

Now the worker is only partially compensated for the OTJ search costs and compensates for the cost of vacancies if OTJ search takes place. Similarly, equations (32b) and (33b) can be used to derive the same wage equations at the highest productivity as in the simple model:

$$\bar{w}_1 = \left(1 - \beta\right) b + \beta \left(p + p c \theta\right) \tag{37}$$

Substituting it into the wage differential, yields the general wage equation:

$$\bar{w}_i = \left(1 - \beta\right) \left(b + e_i \nu\right) + \beta \left(p x_i + \left(1 - e_i\right) p c \theta\right). \tag{38}$$

Its interpretation is the same as in the simple model: the worker is compensated for potentially incurred OTJ search costs and foregoing unemployment benefits and she gets her share of the output and the cost of vacancies if she does not search.

Before I determine the thresholds for OTJ search and job destruction, I can calculate the expressions for the surplus using equations (32a) and (35) as well as (31b) and (33b):

$$S_1 - S_i = \frac{p \left(1 - x_i\right) + e_i \left(p c \theta + \nu\right)}{r + \lambda + e_i a \left(\theta\right)}, \tag{39}$$

$$S_1 = \frac{p - b + \lambda \sum_{j=1}^{n} g\left(x_j\right) \max\left\{S_j, 0\right\} + p c \theta}{r + \lambda + a\left(\theta\right)}. \tag{40}$$

I want to find an equilibrium, in which S_i, and hence W_i and J_i, are monotonic in i. Monotonicity of the surplus implies that there is a threshold R such that jobs are destroyed if $x_i \leq R$. Monotonicity of the worker's share implies that there is a threshold $\bar{S}$ such that OTJ search is performed if $x_i \leq \bar{S}$.[11]

[10] The assumption of continuous time is not crucial for this result. In discrete time, if the worker can either hide her OTJ search efforts from the firm, or take this decision after the wage is set, the same argument applies. Paying a higher wage would not prevent her from performing OTJ search to find a better job for the next period. The firm would have to be able to commit to the wage paid in the next period to prevent OTJ search. This is the case in Shimer (2006), where the wage remains fixed for the duration of the match after it has been initially bargained.

[11] It could be that a non-monotonic surplus is reconcilable with a non-monotonic OTJ search decision because of the existence of multiple equilibria, as shown below. This would create a further inefficiency of the market

The worker will want to perform OTJ search if the expected gains from finding a better job outweigh her OTJ search costs $(a(\theta)(W_1 - W_i) \geq \nu)$. This yields the condition for OTJ search:

$$a(\theta)\beta\frac{p(1-x_i)+e_i(pc\theta+\nu)}{r+\lambda+e_i a(\theta)} \geq \nu. \tag{41}$$

For the individual decision to be consistent with the equilibrium outcome e_i, the condition must hold when $e_i > 0$ and the opposite must hold when $e_i < 1$. For the corner solutions, this yields the two conditions

$$a(\theta)\beta\frac{p(1-x_i)+pc\theta+\nu}{r+\lambda+a(\theta)} \geq \nu, \tag{42}$$

when there is OTJ search and

$$a(\theta)\beta\frac{p(1-x_i)}{r+\lambda} \leq \nu, \tag{43}$$

when there is no OTJ search. The first inequality defines a threshold such that OTJ search is an equilibrium strategy when $x_i \leq \bar{S}_1(\theta)$:

$$\bar{S}_1(\theta) = 1 + c\theta - \frac{r+\lambda+(1-\beta)a(\theta)}{a(\theta)\beta p}\nu. \tag{44}$$

The second inequality defines a threshold such that no OTJ search is an equilibrium strategy when $x_i \geq \bar{S}_o(\theta)$:

$$\bar{S}_0(\theta) = 1 - \frac{r+\lambda}{a(\theta)\beta p}\nu. \tag{45}$$

If $\bar{S}_0(\theta) = \bar{S}_1(\theta)$ held, there would be a unique OTJ search strategy (almost everywhere). However, if $\bar{S}_0(\theta) < \bar{S}_1(\theta)$ holds, there are multiple equilibria for $x_i \in \left[\bar{S}_0(\theta), \bar{S}_1(\theta)\right]$: if nobody is searching in equilibrium, this implies a higher wage, and it is optimal not to search. But if everyone is searching, the wage is lower, and it is indeed optimal for the worker to search. This is the case that will generally happen as $\bar{S}_0(\theta) < \bar{S}_1(\theta)$ is equivalent to

$$\beta pc\theta > (1-\beta)\nu. \tag{46}$$

This condition holds in equilibrium, because from the zero profit condition and the Nash sharing rule it follows that the worker's surplus at the best job is $\frac{\beta}{1-\beta}\frac{pc}{q(\theta)}$. The worker's gain from finding a new job must be smaller than this: As her current job is not destroyed, there is a positive surplus, which reduces the gains from finding a new job. Therefore, the worker's gain from OTJ search is less than $\frac{\beta}{1-\beta}pc\theta$. In contrast, the worker's gain must be larger than her cost ν. Hence, condition (46) holds in any equilibrium, in which OTJ search takes place. Then there are multiple equilibria if $\bar{S}_0(\theta) < 1$ holds, which is equivalent to positive OTJ search costs $(\nu > 0)$. The threshold for OTJ search must thus be in the interval

$$\bar{S} \in \left[1 - \frac{r+\lambda}{a(\theta)\beta p}\nu, 1 + c\theta - \frac{r+\lambda+(1-\beta)a(\theta)}{a(\theta)\beta p}\nu\right]. \tag{47}$$

outcome. But as such an equilibrium seems arbitrary, I exclude it in the further analysis.

For now, I do not specify which equilibrium threshold shall be chosen. But the next subsection shows that all possible thresholds are larger than the efficient one.

A job is destroyed if its surplus becomes negative. I restrict myself to the cases in which there is at least some OTJ search as in the social planner's discussion.[12] Using equations (39) when $e_i = 1$ and (40), it follows that the surplus of a match becomes negative if

$$-px_i + \nu \geq -b + \lambda \sum_{j=1}^{n} g(x_j) \max\{S_j, 0\}. \tag{48}$$

Using equation (40) and the no profit condition, the expected surplus after a λ-shock can be determined:

$$\lambda \sum_{j=1}^{n} g(x_j) \max\{S_j, 0\} = (r + \lambda + a(\theta)) \frac{1}{1-\beta} \frac{pc}{q(\theta)} - p + b - pc\theta. \tag{49}$$

Combining the last two equations, gives the condition for job destruction:

$$p(1 - x_i) + \nu \geq (r + \lambda) \frac{1}{1-\beta} \frac{pc}{q(\theta)} + \frac{\beta}{1-\beta} pc\theta. \tag{50}$$

It is monotonous in x and thus defines the job destruction threshold

$$R(\theta) = 1 - \frac{r+\lambda}{1-\beta} \frac{c}{q(\theta)} - \left(\frac{\beta}{1-\beta} c\theta - \frac{\nu}{p} \right), \tag{51}$$

and the respective dummy variable for job destruction

$$d_i = \begin{cases} 1 & \text{if } x_i \leq R(\theta) \\ 0 & \text{if } x_i > R(\theta) \end{cases}. \tag{52}$$

To find the equilibrium level of market tightness, the analogous job creation condition to equation (18) has to be derived. Combining equations (33b) and (39), making use of the Nash sharing rule, one obtains

$$(r + \delta) J_1 = p - \bar{w}_1 - \lambda \sum_{j=1}^{n} g(x_j) \frac{(1-\beta)[p(1-x_j) + e_j(pc\theta + \nu)]}{r + \lambda + e_j a(\theta)} (1 - d_j), \tag{53}$$

where $\delta \equiv \lambda \sum_{j=1}^{n} g(x_j) I(J_j \leq 0)$ is the probability of job destruction in equilibrium. Substituting the zero profit condition and the wage equation (37), yields the condition

$$\begin{aligned} (r + \delta) \frac{pc}{q(\theta)} &= (1-\beta)(p-b) - \beta pc\theta \\ &\quad - \lambda \sum_{j=1}^{n} g(x_j) \frac{(1-\beta)[p(1-x_j) + e_j(pc\theta + \nu)]}{r + \lambda + e_j a(\theta)} (1 - d_j). \end{aligned} \tag{54}$$

[12] This assumption is even less restrictive here because there will be more OTJ search than in the efficient case.

Using the thresholds for OTJ search and job destruction, this equation implicitly defines labour market tightness in the decentralised equilibrium.

3.3 Comparison of the decentralised equilibrium with the efficient outcome

In this subsection, I show that the market outcome is in general inefficient regardless of the level of bargaining power. This stems from OTJ search taking place in excess, as the worker does not fully incorporate the loss for a firm if she changes the job. Recall the OTJ search thresholds derived in equations (26) and (47):

$$\bar{S}^{SP}(\theta) = 1 - \frac{(pc\theta + \nu)(r + \lambda)}{a(\theta)\, p}, \tag{55a}$$

$$1 - \frac{r + \lambda}{a(\theta)\,\beta p}\nu \leq \bar{S}^{DEC}(\theta) \leq 1 + c\theta - \frac{r + \lambda + (1-\beta)\, a(\theta)}{a(\theta)\,\beta p}\nu. \tag{55b}$$

An efficient search threshold would only be possible if the lower bound for $\bar{S}^{DEC}$ was lower than $\bar{S}^{SP}$, which is equivalent to

$$\beta pc\theta \leq (1-\beta)\,\nu \tag{56}$$

But in (46), exactly the opposite inequality was shown to hold in equilibrium. As a result, if market tightness in the decentralised economy was at the efficient level, the OTJ search threshold would be higher in the decentralised economy and too much OTJ search would be taking place. As this holds regardless of the worker's bargaining power, one can conclude that the outcome in this economy will generally differ from the efficient outcome.[13]

Proposition 2 *The decentralised outcome is not efficient: Conditional on the level of market tightness, there is too much on-the-job search taking place.*

The reason for this result is that without wage commitment, a worker will perform OTJ search whenever the expected gain from finding a better job exceeds her search cost ν. In particular, in the absence of OTJ search costs there will be OTJ search for all but the highest level of productivity. From a social point of view, the gain of finding a better job is smaller as the old job is destroyed and the firm loses its part of the rent. Even in the absence of OTJ search costs, the social planner's threshold (26) shows that not all workers should seek better jobs. If the level of productivity is close to the optimal, it is not worth maintaining the additional vacancies to potentially find a better job for the worker. This result differs from Pissarides (2000) and Stevens (2004) who both conclude that there is too little search taking place. First, Pissarides (2000) assumes that the decision for OTJ search is taken jointly by the firm and the worker such that their surplus is maximised. In this section however, there is no commitment and the worker will base the OTJ search decision on her individual optimality conditions. Second, they

[13] As I assume discrete levels of productivity, the efficient outcome can be achieved if there is no possible level of productivity between the two different thresholds. In general, the difference between the two outcomes will be determined by the probability of a match having a productivity level between the efficient and the decentralised threshold.

do not take into account or do not model the cost of maintaining vacancies for OTJ search. In particular, in the absence of (direct) OTJ search costs, the efficient level of OTJ search is that any worker with less than optimal productivity performs OTJ search. This does not take into account that either more vacancies are needed to maintain the level of market tightness or there is a congestion effect decreasing the chances of other job seekers who need a (better) job more strongly.

The second decision of the agents in the economy that could potentially differ from the efficient level, is job destruction. Nash sharing ensures that this decision is jointly optimal and there is no (direct) distortion. In other words, a match will be destroyed endogenously whenever its surplus vanishes. This is the same action a social planner would take, conditional on the OTJ search threshold and market tightness.

In the next section, I allow for (partial) commitment, which can lead to an efficient OTJ search decision and the first best outcome.

4 Decentralised equilibrium with (partial) commitment

4.1 Setup

I extend the model from above to allow for partial commitment in terms of wages: I assume that, unless productivity changes, the firm and the worker can only renegotiate the wage after a shock with arrival rate ω has arrived. The parameter ω represents the level of commitment, such that if it is 0, there is perfect commitment (until a productivity shock arrives). As ω becomes larger, the two parties are allowed to renegotiate after increasingly shorter intervals. The intuitive consequence of wage commitment is that it becomes feasible for the firm to prevent OTJ search by paying and committing to a higher wage. Without commitment, it was shown that there is too much OTJ search taking place as the worker does not care about the loss the firm incurs. Below, I show that for any finite level of ω, it is possible to achieve the efficient OTJ search threshold in equilibrium. This result hinges on choosing an appropriate subgame perfect equilibrium in the bargaining game over a non-convex bargaining set.

Denote by $W_i(\bar{w}_i)$ and $J_i(\bar{w}_i)$ the worker's and the firm's value of a match when the wage is fixed at $\bar{w}_i$. The equilibrium values that are expected after renogatiation are denoted by W_i and J_i, respectively. Adapting the stationary value equations then yields

$$\begin{aligned} rW_i(\bar{w}_i) &= \bar{w}_i + \lambda \sum_{j=1}^{n} g(x_j)\left(\max\{W_j, U\} - W_i(\bar{w}_i)\right) \\ &\quad + e_i\left[a(\theta)\left(W_1 - W_i(\bar{w}_i)\right) - \nu\right] + \omega\left(W_i - W_i(\bar{w}_i)\right), \end{aligned} \tag{57}$$

$$rJ_i(\bar{w}_i) = px_i - \bar{w}_i + \lambda \sum_{j=1}^{n} g(x_j)\left(\max\{J_j, 0\} - J_i(\bar{w}_i)\right) - e_i a(\theta) J_i(\bar{w}_i) + \omega\left(J_i - J_i(\bar{w}_i)\right). \tag{58}$$

The worker's optimal search decision is $e_i = 1$ if $a(\theta)\left(W_1 - W_i(\bar{w}_i)\right) > \nu$ such that the expected gain exceeds her search cost.

Before the outcome of the bargaining process is analysed in detail, note that in an equilibrium, in which everyone sets the same wage, the level of commitment ω will not directly influence the wage. It will rather influence the OTJ search decision. It was shown above that there might be multiple equilibria without commitment: there can be an equilibrium featuring a low wage and OTJ search that is Pareto-dominated by an equilibrium with a higher wage and no OTJ search. Since the lack of commitment does not make it feasible for the firm to increase the wage by as much as is needed to prevent the worker from performing OTJ search, these bad equilibria could not be ruled out. Furthermore, new equilibria are feasible, when the bargaining game is adjusted to the non-convex bargaining set.

The bargaining set becomes potentially non-convex, because raising the wage can prevent the worker from OTJ search and make both the firm and the worker better off. To see this formally, the dependence of the worker's and the firm's surplus on the bargained wage can be analysed. The worker's value equation can be rewritten as

$$W_i\left(\bar{w}_i\right)=\frac{\bar{w}_i+e_i\left[a\left(\theta\right)W_1-\nu\right]+c_i^W}{r+\lambda+\omega+e_i a\left(\theta\right)}, \tag{59}$$

where $c_i^W\equiv\lambda\sum_{j=1}^n g\left(x_j\right)\max\left\{W_j,U\right\}+\omega W_i$ does not depend on the outcome of the bargaining. The worker's surplus is strictly increasing in the bargained wage $\bar{w}_i$. The threshold $\bar{w}_i^{NS}$ for OTJ search is given by the condition $a\left(\theta\right)\left(W_1-W_i\left(\bar{w}_i^{NS}\right)\right)=\nu$ which yields:

$$\bar{w}_i^{NS}=\left(r+\lambda+\omega\right)\left(W_1-\frac{\nu}{a\left(\theta\right)}\right)-c_i^W. \tag{60}$$

The worker's optimal choice of this threshold ensures that her value function is continuous at $\bar{w}_i^{NS}$. But there is a kink as the slope for lower wages (i.e. when there is OTJ search) is $\frac{1}{r+\lambda+\omega+a(\theta)}$ which is less than the slope for higher wages ($\frac{1}{r+\lambda+\omega}$, when there is no OTJ search). Hence, the worker's surplus is continuous, piecewise linear, increasing, and convex in her wage. Using the condition $W_i\left(\bar{w}_i^R\right)=U$, her reservation wage $\bar{w}_i^R$ is given by:

$$\begin{aligned}\bar{w}_i^R &= \left(r+\lambda+\omega\right)U-e_i\left[a\left(\theta\right)\left(W_1-U\right)-\nu\right]-c_i^W\\ &= rU-e_i\left[a\left(\theta\right)\left(W_1-U\right)-\nu\right]-\lambda\sum_{j=1}^n g\left(x_j\right)\max\left\{W_j-U,0\right\}-\omega\left(W_i-U\right). \end{aligned} \tag{61}$$

Likewise, the firm's surplus is given by:

$$\left(r+\lambda+\omega+e_i a\left(\theta\right)\right)J_i\left(\bar{w}_i\right) = px_i-\bar{w}_i+\lambda\sum_{j=1}^n g\left(x_j\right)\max\left\{J_j,0\right\}+\omega J_i, \tag{62a}$$

$$J_i\left(\bar{w}_i\right) = \frac{-\bar{w}_i+c_i^J}{r+\lambda+\omega+e_i a\left(\theta\right)}, \tag{62b}$$

where the maximum wage that the firm is willing to pay, does not depend on the outcome of the bargaining:

$$c_i^J = px_i + \lambda \sum_{j=1}^{n} g(x_j) \max\{J_j, 0\} + \omega J_i. \tag{63}$$

The employer's surplus from bargaining is decreasing in the wage, piecewise linear, but not continuous at $\bar{w}_i^{NS}$: the limit from below (i.e. when there is OTJ search) is $\frac{-\bar{w}_i^{NS}+c^J}{r+\lambda+\omega+a(\theta)}$, whereas the limit from above is higher ($\frac{-\bar{w}_i^{NS}+c^J}{r+\lambda+\omega}$). This discontinuity reflects that at one side of the threshold the employer risks to lose the worker due to OTJ search but marginally increasing her wage prevents OTJ search and thus increases the expected job duration. Denote by $\bar{w}_i^S < \bar{w}_i^{NS}$ the maximum wage the firm is willing to pay when there is OTJ search without being better off by raising the wage to $\bar{w}_i^{NS}$. Using the firm's surplus in equation (62b), this condition

$$J_i\left(\bar{w}_i^S \,|e_i = 1\right) = J_i\left(\bar{w}_i^{NS} \,|e_i = 0\right) \tag{64}$$

yields

$$\bar{w}_i^S = (r + \lambda + \omega + a(\theta)) \left(W_1 - \frac{\nu}{a(\theta)}\right) - c_i^W - \frac{a(\theta)\left(c_i^W + c_i^J\right)}{r + \lambda + \omega}. \tag{65}$$

Therefore no wage in the interval $\left(\bar{w}_i^S, \bar{w}_i^{NS}\right)$ should be the outcome of the bargaining process as both the firm and the worker could do better by raising the wage to $\bar{w}_i^{NS}$.

Appendix A.2 shows that it is jointly optimal to perform OTJ search if the following condition holds:

$$a(\theta)(W_1 - W_i(\bar{w}_i) - J_i(\bar{w}_i)) \geq \nu. \tag{66}$$

This is a stronger condition than the worker's individual optimality condition:

$$a(\theta)(W_1 - W_i(\bar{w}_i)) \geq \nu, \tag{67}$$

This is, because the worker does not take into account the firm's loss if she finds a new job. Hence, if it is jointly optimal to perform OTJ search, it will also be individually rational to do so for the worker at any wage that is feasible for the firm.[14] The outcome of the bargaining process will not influence the OTJ search decision and the firm and the worker bargain over the surplus S_i^S. Nash bargaining can be applied to determine the wage in this case as in the previous sections. If on the other hand it is not jointly optimal to perform OTJ search, it is still individually rational for the worker to do so if the wage is below $\bar{w}_i^{NS}$. Then, the bargaining set becomes non-convex and standard Nash bargaining cannot be applied. Similar to Shimer (2006), I use an alternating offer bargaining game to determine the wage in this case. It is described and solved in the next subsection.

In contrast to the case of no wage commitment, the bargaining surplus depends on the search decision implied by the bargained wage. In the previous section, it did not matter, because the wage was renegotiated after search, making the search decision only dependent on the equilibrium wage, which is taken as given. To see the dependence on ω, the difference between the surplus

[14] Formally, this means that $c_i^J \leq \bar{w}_i^{NS}$ holds. Then, the worker will search even if the firm offers the maximum wage such that its rent becomes 0.

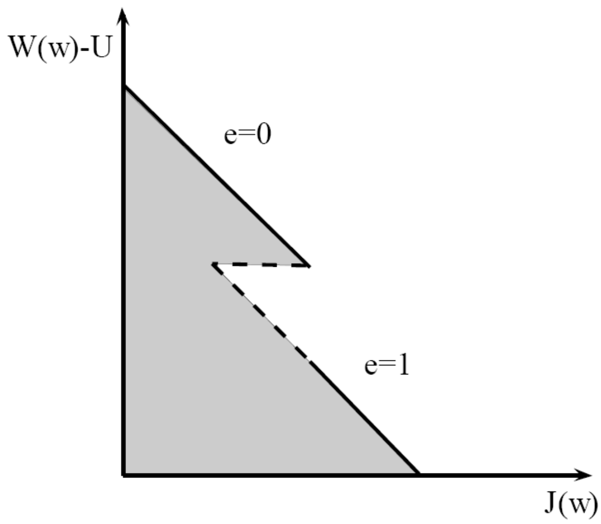

Figure 1: Non-convex bargaining set when OTJ search does not maximise the joint surplus: When the worker's share is sufficiently high, she does not perform OTJ search, which maximises $W - U + J$. Below the cutoff, the worker searches OTJ and the firm's value drops discontinuously. The solidly drawn frontier of the bargaining set can be obtained in an alternating bargaining game in Lemma 3.

with and without OTJ search can be calculated from equations (86) and (87):

$$S_i^{NS} - S_i^S = \frac{a(\theta)}{r + \lambda + \omega + a(\theta)} \left[\frac{-p(1 - x_i) + (r + \lambda) S_1 + \omega S_i}{r + \lambda + \omega} - \left(W_1 - U - \frac{\nu}{a(\theta)} \right) \right]. \quad (68)$$

As $\omega \to \infty$, which corresponds to the case of no commitment, the term in brackets is bounded. Therefore, the difference between S_i^S and S_i^{NS} vanishes in the limit. This proves that, as the wage is renegotiated more frequently, the bargained wage becomes less important and the wages in equilibrium dominate the worker's OTJ search decision.

The next subsection describes the bargaining game used to deal with the potential non-convexity.

4.2 Wage bargaining when the bargaining set is non-convex

An example for the non-convex bargaining set when OTJ search is socially inefficient is depicted in Figure 1.This set can in general be described as follows. Denote by $x \in [0, S]$ the worker's surplus and by x^{NS} the threshold at which the worker's search behaviour changes. Let T be the joint loss when there is OTJ search. Then the firm's surplus is given by

$$J(x) = \begin{cases} S - x & \text{if } x^{NS} \le x \le S \\ S - x - T & \text{if } 0 \le x < x^{NS} \end{cases}. \quad (69)$$

Following Shimer (2006), I consider an alternating offer bargaining game extending the Rubinstein (1982) model: in each stage of the bargaining process the worker is allowed to make an offer with probability $\beta \in [0,1]$ and the firm makes an offer with probability $1-\beta$.[15] If the receiver of the offer accepts, bargaining ends with the respective payoffs. If the offer is rejected, the bargaining process breaks down with probability $1-\delta$ before the next stage.

Lemma 3 *When δ is approaching 1, the following values for the worker's surplus can be obtained in a subgame perfect equilibrium:*

- $x = \beta S$ *if* $\beta > \frac{x^{NS}}{S}$
- $x = x^{NS}$ *if* $\beta \leq \frac{x^{NS}}{S}$
- $x = \beta (S - T)$ *if* $\beta < \frac{x^{NS}-T}{S-T}$

Proof. See Appendix A.3 ■

Note that there are multiple equilibria if $\beta < \frac{x^{NS}-T}{S-T}$ holds: both an equilibrium without OTJ search (the surplus maximizing outcome) as well as one with OTJ search and Nash sharing could be obtained. Even low β, this is a high bargaining power of the firm, cannot break the surplus maximizing equilibrium, because potential marginal gains for the firm are offset by a discrete loss for the worker. Therefore, as δ approaches 1, the worker would not accept such an offer, even if she potentially has to wait a long time until making an offer again.

The selection of one of the multiple equilibria when $\beta < \frac{x^{NS}-T}{S-T}$ determines the efficieny of the decentralised solution. choice is to be compared to the solution of a social planner. In the following section, I choose the equilibrium that leads to an efficient OTJ search decision. This means that $x = x^{NS}$ is chosen for all $\beta \leq \frac{x^{NS}}{S}$. The worker's surplus is large enough to prevent her from performing OTJ search. What makes this equilibrium attractive is that it maximises the joint surplus and therefore yields an optimal search decision given market tightness. At the same time, it also maximises the worker's share that does not fall below $\frac{x^{NS}}{S}$.[16]

To conclude the discussion of a non-convex bargaining set, note that mixed lotteries could potentially Pareto improve the outcome: by mixing over $x = 0$ and $x = x^{NS}$, both agents can be made better off compared to the equilibria in cases 2 and 3 above, when $\beta < \frac{x^{NS}}{S}$. In this case, the bargaining set becomes convex and Nash bargaining can be applied. Following Shimer (2006), I rule out this possibility, as a wage lottery being the outcome of a bargaining process might be difficult to interpret and implement.

4.3 Determining the equilibrium

In section 4 above, the reservation wage $\bar{w}_i^R$, the maximum wage c_i^J, the maximum wage with OTJ search $\bar{w}_i^S$, and the minimum wage without OTJ search $\bar{w}_i^{NS}$ were determined in equations (61), (63), (65), and (60), respectively. These values only depend on the equilibrium outcome but not on the individually bargained wage. It always holds that $\bar{w}_i^S$ is smaller than $\bar{w}_i^{NS}$

[15] β is again interpreted as the worker's bargaining power.

[16] Alternative selections of an equilibrium in the bargaining game could be the equilibrium that maximises the firm's share, i.e. $x = \beta (S - T)$ if $\beta < \frac{x^{NS}-T}{S-T}$, or the one that maximises the weighted product of returns $x^{\beta} J(x)^{1-\beta}$.

but the relative ranking of reservation wage and maximum wage determine the outcome of the bargaining process. I assume that $\bar{w}_i^R \leq \bar{w}_i^{NS}$ holds such that OTJ search takes place at some levels of productivity in equilibrium.[17] Three cases have to be distinguished:

1. Job destruction zone ($\bar{w}_i^R > c_i^J$): if the reservation wage is larger than the maximum wage the firm is willing to pay, the result is immediate job destruction. When OTJ search takes place for low levels of productivity, jobs are destroyed if their output plus the option value of getting hit by a λ-shock is less than unemployment benefit plus OTJ search costs. This is the same condition as in the case without commitment.

2. On-the-job search zone ($\bar{w}_i^R \leq c_i^J < \bar{w}_i^{NS}$): this is the case, in which it is jointly optimal for the worker and the firm and also individually rational for the worker to perform OTJ search. Therefore, the worker will perform OTJ search after any outcome of the bargaining process and Nash bargaining can be used again. Using the above definitions, the condition $c_i^J < \bar{w}_i^{NS}$ can be rearranged, yielding

$$a(\theta)(W_1 - W_i - J_i) > \nu. \tag{70}$$

   This means exactly that the joint benefit from OTJ search is higher than the cost.

3. Non-convex bargaining set ($\bar{w}_i^{NS} \leq c_i^J$): it would be jointly optimal not to perform OTJ search ($a(\theta)(W_1 - W_i - J_i) \leq \nu$) but for low values of the wage it is individually rational for the worker to do so ($\bar{w}_i^R \leq \bar{w}_i^{NS}$). Therefore, the bargaining set becomes non-convex. In this case, I apply the bargaining process from above to determine the wage. In the notation of section 4.2, $S \equiv S_i^{NS}$, $T \equiv S_i^{NS} - S_i^S$, and $x^{NS} \equiv W_i\left(\bar{w}_i^{NS}\right) - U = W_1 - U - \frac{\nu}{a(\theta)}$. Applying Lemma 3 and choosing the equilibrium as discussed above, the worker's surplus is

$$\begin{aligned} W_i - U &= \begin{cases} \beta S_i^{NS} & \text{if } \beta S_i^{NS} \geq W_1 - U - \frac{\nu}{a(\theta)} \\ W_1 - U - \frac{\nu}{a(\theta)} & \text{else} \end{cases} \\ &= \max\left(\beta S_i^{NS}, W_1 - U - \frac{\nu}{a(\theta)}\right). \end{aligned} \tag{71}$$

Having determined the individual behaviour in the bargaining process, I can find the outcome in equilibrium, where $W_i(\bar{w}_i) = W_i$ and $J_i(\bar{w}_i) = J_i$ hold. Then the value equations for given $\bar{w}_i$ and e_i are the same as in section 3.2 regardless of the level commitment. In particular, also the surplus is given by equations (39) and (40).

Appendix A.4 shows that the same threshold for OTJ search as a function of market tightness as in the social planner's problem is obtained. The firing threshold is also efficient conditional on market tightness. Depending on the level of productivity, the worker's surplus and optimal

[17] The condition $w_i^R \leq w_i^{NS}$ is equivalent to $a(\theta)(W_1 - U) \geq \nu$ as the reservation wage reduces the worker's value to U and she will only want to search if the option value of finding a job is larger than the cost. If it did not hold, the OTJ-search costs are prohibitively large and there could never be OTJ-search as $W_i \geq U$ holds.

decision are summarised in the following table:

Decision	$W_i - U$	Range of productivity
No OTJ search	βS_i^{NS}	$x\in\left[1-\frac{r+\lambda}{a(\theta)\beta p}\nu,1\right]$
No OTJ search	$\beta S_1^{NS} - \frac{\nu}{a(\theta)}$	$x\in\left[1-\frac{r+\lambda}{a(\theta)p}(\nu+pc\theta),1-\frac{r+\lambda}{a(\theta)\beta p}\nu\right)$
OTJ search	βS_i^S	$x\in\left[1-\frac{r+\lambda}{1-\beta}\frac{c}{q(\theta)}-\left(\frac{\beta}{1-\beta}c\theta-\frac{\nu}{p}\right),1-\frac{r+\lambda}{a(\theta)p}(\nu+pc\theta)\right)$
Job destruction	0	$x\in\left[0,1-\frac{r+\lambda}{1-\beta}\frac{c}{q(\theta)}-\left(\frac{\beta}{1-\beta}c\theta-\frac{\nu}{p}\right)\right)$

Given the worker's surplus at all levels of productivity, her wage can be calculated from equation (57):

$$\bar{w}_1 = b + (r + \lambda + a(\theta)) \frac{\beta}{1-\beta} \frac{pc}{q(\theta)} + \lambda \sum_{j=1}^{n} g(x_j) \max\{W_j - U, 0\}. \tag{72}$$

As the surplus is not split in the same proportion at all levels any more, the expression for $\bar{w}_1$ becomes more complicated. The wage differential, however, can still be easily obtained:

$$\bar{w}_1 - \bar{w}_i = (r+\lambda)(W_1 - U) - (r + \lambda + e_i a(\theta))(W_i - U) + \tfrac{\beta}{1-\beta} e_i pc\theta - e_i \nu \tag{73}$$

$$= \begin{cases} \beta p (1 - x_i) & \text{No OTJ search} \\ (r+\lambda) \frac{\nu}{a(\theta)} & \text{Corner Solution} \\ \beta p (1 - x_i) + \beta pc\theta - (1-\beta)\nu & \text{OTJ search} \end{cases}. \tag{74}$$

The wage differential is the same as before in the first and the third case, as the surplus is still shared according to the Nash rule. In the intermediate case, when the worker is just made indifferent between searching and not, the differential reflects exactly this indifference: her expected cost of finding a job that pays $\bar{w}_1$ is $\frac{\nu}{a(\theta)}$ whereas the expected loss from the lower wage is $\frac{\bar{w}_1 - \bar{w}_i}{r+\lambda}$.

The main result of this section is summarised in the following proposition:

Proposition 4 *The efficient threshold for on-the-job search conditional on market tightness can be obtained at any positive level of wage commitment.*

4.4 Comparison with the no-commitment case

The result in proposition 3 is a consequence of the non-convex bargaining set that is obtained whenever OTJ search is not efficient. Of course, it crucially depends on the choice of the equilibrium in the bargaining game: even if the worker's bargaining power and the possible degree of inefficiency is small, the equilibrium at the kink of the bargaining set was chosen. On the one hand, this is a strong assumption when the worker and the firm enter such a bargaining game after a shock. On the other hand, it maximises the expected surplus when the job is created. The firm is thus compensated by having to pay a lower wage before such a shock, in anticipation of possible future shocks.

Finally, I want to give intuition why the limit for $\omega \to \infty$ does not give the results obtained in the section without commitment. It is useful to display the differences in a discrete time setup that yields the continuous time model in the limit. In this model, the timing is as follows (abstracting from the arrival of λ-shocks):

1. The wage is determined for this period.
2. Worker makes OTJ search decision.
3. Next period: wage is renegotiated with probability $1 - e^{-\omega}$

When the wage is renegotiated for sure, only the equilibrium values of the payoffs matter for the continuation value in the third stage. Hence the worker's OTJ search decision does not depend on her current wage and the bargaining set is convex. Nash bargaining then determined the wage. For a finite value ω, however, the continuation value can be influenced by the current wage and the bargaining set might become non-convex. The chosen equilibrium implied that the worker's share is higher than β in some cases to prevent her from OTJ search. If this implicit non-constant bargaining power was also used in the Nash bargaining in the no-commitment case, the efficient OTJ search decision could also be obtained.

5 Conclusion

In this paper, I have discussed two possible inefficiencies that are present in a search model with OTJ search. On the one hand, opening vacancies imposes externalities on other firms and job seekers. It was shown that the worker's bargaining power as suggested by the Hosios rule is too low, because of OTJ search. On the other hand, a worker that quits her firm imposes an externality on it. In general, this leads to too much job turnover. It was shown that allowing for wage commitment can help to reduce this externality. Using a suitable subgame perfect equilibrium the efficient OTJ search decision could be obtained.

As pointed out above, the choice of the respective equilibrium is crucial for this result. In an extension of this paper, the consequences of different bargaining mechanisms may be analysed. A second limitation of this paper is that it focuses only on the outcome in a steady state. Compared to Menzio and Shi (2011), it is more difficult to study transitional dynamics; for example, the distribution of matches over productivity influences the efficient level of labour market tightness. In the presence of aggregate shocks, this distribution will depend on the past history of shocks, because the thresholds for destruction and OTJ search become time-varying.

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A Appendix

A.1 Solution of Social Planner's problem in the full model

The costates are denoted by U and W_i, respectively, which yields the Hamiltonian in the current-value form:

$$\begin{aligned} H &= ub + p\sum_{i=1}^{n} x_i w_i - pc\theta\left(u + \sum_{i=1}^{n} e_i w_i\right) - \nu \sum_{i=1}^{n} e_i w_i \\ &+U\left[\lambda \sum_{i=1}^{n} d_i g\left(x_i\right) \sum_{j=1}^{n} w_j - a\left(\theta\right) u\right] + \\ &+W_1\left[\lambda\left(g\left(x_1\right)\sum_{j=1}^{n} w_j - w_1\right) + a\left(\theta\right)\left(u + \sum_{i=1}^{n} e_i w_i\right)\right] + \\ &+\sum_{i=2}^{n} W_i\left[\lambda\left(\left(1-d_i\right) g\left(x_i\right)\sum_{j=1}^{n} w_j - w_i\right) - e_i a\left(\theta\right) w_i\right]. \end{aligned} \quad (75)$$

By differentiating with respect to θ, the analogous first order condition to equation (3b) is obtained:

$$a'\left(\theta\right)\left[u\left(W_1 - U\right) + \sum_{i=1}^{n} e_i w_i \left(W_1 - W_i\right)\right] = pc\left(u+e\right), \quad (76a)$$

$$\left(1-\eta\right)\left[\frac{u}{u+e}\left(W_1 - U\right) + \sum_{i=1}^{n} \frac{e_i w_i}{u+e}\left(W_1 - W_i\right)\right] = \frac{pc}{q\left(\theta\right)}. \quad (76b)$$

where $e \equiv \sum_{i=1}^{n} e_i w_i$ denotes the mass of OTJ seekers. Again, the marginal cost of increasing θ must equal the marginal benefit from matching more unemployed and employed workers where the weighted average is now over all job seekers.

Maximisation with respect to e_i gives the condition for OTJ search

$$a\left(\theta\right)\left(W_1 - W_i\right) \geq pc\theta + \nu. \quad (77)$$

OTJ search takes place at productivity x_i, if the expected gain from improving the productivity outweighs the cost of opening additional vacancies and of the OTJ search cost.

A match is destroyed, if the value of an unemployed agent becomes larger than the value of keeping the job:

$$U \geq W_i. \quad (78)$$

The differential equations for the costates become:

$$\dot{U} = rU - [b - pc\theta + a(\theta)(W_1 - U)], \tag{79a}$$

$$\dot{W}_1 = rW_1 - \left[p + \lambda\left(\sum_{j=1}^{n} g(x_j)[d_j U + (1-d_j) W_j] - W_1\right)\right], \tag{79b}$$

$$\begin{aligned}\dot{W}_i &= rW_i - [px_i + e_i[a(\theta)(W_1 - W_i) - pc\theta - \nu]] \\ &\quad -\lambda\left(\sum_{j=1}^{n} g(x_j)[d_j U + (1-d_j) W_j] - W_i\right),\end{aligned} \tag{79c}$$

and in a steady state the analogous equations to (5a), (5b), and (5c) are obtained:

$$rU = b - pc\theta + a(\theta)(W_1 - U), \tag{80a}$$

$$rW_1 = p + \lambda\left(\sum_{j=1}^{n} g(x_j)[d_j U + (1-d_j) W_j] - W_1\right), \tag{80b}$$

$$\begin{aligned}rW_i &= px_i + e_i[a(\theta)(W_1 - W_i) - pc\theta - \nu] \\ &\quad +\lambda\left(\sum_{j=1}^{n} g(x_j)[d_j U + (1-d_j) W_j] - W_i\right).\end{aligned} \tag{80c}$$

Subtracting the equations from each other, one can obtain an expression for the respective surplus:

$$\begin{aligned}(r + e_i a(\theta) + \delta)(W_i - U) &= px_i - e_i\nu - b - (1-e_i)(a(\theta)(W_1 - U) - pc\theta) \\ &\quad +\lambda\sum_{j=1}^{n}(1-d_j) g(x_j)(W_j - W_i),\end{aligned} \tag{81a}$$

$$(r + e_i a(\theta) + \lambda)(W_1 - W_i) = p(1 - x_i) + e_i(pc\theta + \nu), \tag{81b}$$

$$(r + a(\theta) + \delta)(W_1 - U) = p - b + pc\theta + \lambda\sum_{j=1}^{n}(1-d_j) g(x_j)(W_j - W_1). \tag{81c}$$

where $\delta \equiv \lambda \sum_{i=1}^{n} d_i g(x_i)$ denotes the (endogenous) rate of job destruction. The second set of equations could be substituted into the last to obtain $W_1 - U$ depending only on the control variables.

Substituting equation (81b) into the OTJ search condition (77) yields

$$a(\theta)\frac{p(1-x_i)}{r+\lambda} \geq pc\theta + \nu, \tag{82}$$

which is monotonous in x_i. Hence, for each θ there exists a threshold $\bar{S}(\theta)$, such that there is OTJ search if $x \leq \bar{S}(\theta)$:

$$\bar{S}(\theta) = 1 - \frac{(pc\theta + \nu)(r + \lambda)}{a(\theta)p}. \tag{83}$$

A.2 Derivation of jointly optimal OTJ search decison with (partial) commitment

Under partial commitment, the joint value of a match is given by

$$(r+\lambda+\omega)\left(W_i\left(\bar{w}_i\right)+J_i\left(\bar{w}_i\right)\right)=e_i\left[a(\theta)\left(W_1-W_i\left(\bar{w}_i\right)-J_i\left(\bar{w}_i\right)\right)-\nu\right]+c_i^W+c_i^J, \tag{84}$$

where

$$c_i^W+c_i^J=-p\left(1-x_i\right)+(r+\lambda)\left(W_1+J_1\right)+\omega\left(W_i+J_i\right). \tag{85}$$

It is piecewise constant and has a jump discontinuity at $\bar{w}_i^{NS}$: when the worker does not want to search, the joint surplus is

$$S_i^{NS}=\frac{-p\left(1-x_i\right)+(r+\lambda) S_1+\omega S_i}{r+\lambda+\omega}, \tag{86}$$

and when the worker wants to search it is

$$S_i^S=\frac{-p\left(1-x_i\right)+(r+\lambda) S_1+\omega S_i+a(\theta)\left(W_1-U\right)-\nu}{r+\lambda+\omega+a(\theta)}. \tag{87}$$

The ranking of the two determines whether it is jointly optimal for the firm and the worker to perform OTJ search. They should jointly agree on OTJ search if

$$a(\theta)\left(W_1-W_i\left(\bar{w}_i\right)-J_i\left(\bar{w}_i\right)\right) \geq \nu \tag{88}$$

holds.

A.3 Proof of Lemma 3

Proof. Consider the following strategies: when they get the chance, the worker proposes x^W and the firm proposes x^F. The worker accepts any proposal $x \geq x^F$ and the firm accepts if $J(x) \geq J\left(x^W\right)$. I want to find x^W and x^F, such that these strategies constitute a subgame perfect equilibrium. The firm must be indifferent between accepting or not. This condition arises, because if the firm was strictly better off from accepting, the worker could marginally raise her share. This would make the worker better off and the firm at most only marginally worse off. In contrast, it can be the case that the worker is strictly better off from accepting the firm's offer. This difference is caused by the non-convexity: if x^W is lager than $x^{NS}-T$, it is best for the firm to offer at least x^{NS} and thereby prevent the worker from OTJ search. Therefore, the firm will not propose $x^F \in\left(x^{NS}-T, x^{NS}\right)$. This yields the conditions

$$J\left(x^W\right) = \delta\left[\beta J\left(x^W\right)+(1-\beta) J\left(x^F\right)\right], \tag{89a}$$

$$x^F \geq \delta\left[\beta x^W+(1-\beta) x^F\right]. \tag{89b}$$

being equivalent to

$$(1-\delta\beta) J\left(x^{W}\right) = \delta(1-\beta) J\left(x^{F}\right), \tag{90a}$$

$$x^{F} \geq \frac{\delta\beta x^{W}}{1-\delta(1-\beta)}. \tag{90b}$$

The inequality can only be strict if $x^F = x^{NS}$.

First, I discuss the case of equality; it immediately follows that $x^W > x^F$ holds as there is an advantage of being allowed to make an offer. Furthermore, both offers must lie on the same side of the discontinuity at x^{NS}, if δ is sufficiently large: if the worker's offer involved no OTJ search but the firm's offer did, they cannot both be indifferent between accepting or not as the (discounted) surplus is bigger when the worker makes the offer. Substituting x^F into the firm's payoff function yields

$$(1-\delta)\left(S - I\left(x^{W} < x^{NS}\right) T\right) - (1-\delta\beta) x^{W} = -\frac{\delta(1-\beta)\delta\beta}{1-\delta(1-\beta)} x^{W}. \tag{91}$$

Rearranging it, gives the worker's surplus proposed by a worker and consequently by a firm:

$$x^{W} = [1-\delta(1-\beta)]\left(S - I\left(x^{W} < x^{NS}\right) T\right), \tag{92a}$$

$$x^{F} = \delta\beta\left(S - I\left(x^{W} < x^{NS}\right) T\right). \tag{92b}$$

As $\delta \to 1$, the worker's surplus converges to

$$x = \begin{cases} \beta S & \text{if } \beta \geq \frac{x^{NS}}{S} \\ \beta(S-T) & \text{if } \beta < \frac{x^{NS}-T}{S-T} \end{cases}.. \tag{93}$$

The bargained share is such that x^F indeed lies outside the interval $\left(x^{NS}-T, x^{NS}\right)$ for δ sufficiently large. This makes both strategies a best response and a subgame perfect equilibrium is found.

Second, I discuss the alternative where $x^F = x^{NS}$ holds, so that the firm just induces the worker not to perform OTJ search. As the firm must be indifferent between accepting or not, the worker's offer cannot be smaller than x^{NS} as well. Using condition (90a) for the firm's indifference, yields the worker's offer

$$x^{W} = \frac{(1-\delta) S + \delta(1-\beta) x^{NS}}{1-\delta\beta}. \tag{94}$$

As $\delta \to 1$, the worker's surplus converges to $x = x^{NS}$, which is the claimed surplus in equilibrium for $\beta \leq \frac{x^{NS}}{S}$. For it to be a best response by the worker $x^{NS} = x^F \geq \frac{\delta\beta x^W}{1-\delta(1-\beta)}$ has to hold. This gives the condition

$$x^{NS} \geq \frac{\delta\beta}{1-\delta(1-\beta)} \frac{(1-\delta) S + \delta(1-\beta) x^{NS}}{1-\delta\beta}, \tag{95}$$

which is equivalent to

$$\beta \leq \frac{x^{NS}}{\delta S}. \tag{96}$$

Therefore, $x = x^{NS}$ can indeed be obtained as δ approaches 1 for all $\beta \leq \frac{x^{NS}}{S}$. ■

A.4 Derivation of equilibrium with (partial) commitment

Workers in matches with high productivity do not perform OTJ search and the surplus is divided by Nash bargaining. The range of such productivities can be determined using equations (86) and (39) when $e_i = 0$ as well as that the Nash sharing rule holds for the maximum productivity:

$$\beta S_i^{NS} \geq W_1 - U - \frac{\nu}{a(\theta)}, \tag{97a}$$

$$\beta \frac{p(1-x_i)}{r+\lambda} \leq \frac{\nu}{a(\theta)}. \tag{97b}$$

This is the same condition as condition (43) in the case of no commitment that defined the lower bound $S_0(\theta) = 1 - \frac{r+\lambda}{a(\theta)\beta p}\nu$ for the range of productivities not featuring OTJ search in equilibrium.

If productivity is below this threshold, there is still no OTJ search but the wage is not determined by Nash bargaining but using the bargaining process for non-convex sets. The worker's value is then at the corner solution $W_i = W_1 - \frac{\nu}{a(\theta)}$ as long as $a(\theta)(W_1 - W_i - J_i) \leq \nu$ holds, which yields the condition

$$a(\theta)(S_1 - S_i - J_1) \leq \nu, \tag{98a}$$

$$a(\theta)\frac{p(1-x_i)}{r+\lambda} \leq \nu + pc\theta. \tag{98b}$$

This determines the OTJ search threshold $S(\theta) = 1 - \frac{r+\lambda}{a(\theta)p}(\nu + pc\theta)$, which is indeed the threshold obtained in the case of a social planner in equation (26).

Below this threshold, OTJ search will take place until the productivity becomes too small and the job is destroyed. The job is destroyed if $\bar{w}_i^R > c_i^J$, which using equations (85) and (61) yields

$$(r+\lambda+\omega)U - e_i[a(\theta)(W_1 - U) - \nu] > c_i^W + c_i^J, \tag{99a}$$

$$-e_i a(\theta)\left[S_1 - \frac{pc\theta + \nu}{a(\theta)}\right] > -p(1-x_i) + (r+\lambda)S_1 + \omega S_i. \tag{99b}$$

In an equilibrium with at least some OTJ search, $e_i = 1$ and $S_R = 0$ hold, which yields the condition

$$p(1-x_i) + pc\theta + \nu > (r+\lambda+a(\theta))\frac{pc}{(1-\beta)q(\theta)}, \tag{100}$$

and hence the reservation productivity

$$R(\theta) = 1 - \frac{r+\lambda}{1-\beta}\frac{c}{q(\theta)} - \left(\frac{\beta}{1-\beta}c\theta - \frac{\nu}{p}\right). \tag{101}$$

It is exactly the threshold obtained in equation (51) in the case without commitment. It is not surprising that it does not depend on the level of commitment in equilibrium as the firm and the worker jointly decide to destroy the job, which only happens if the surplus is negative.

Using equations (86) and (87) as well as the zero-profit condition one obtains the surplus with and without OTJ search in equilibrium:

$$S_i^S = \frac{pc}{(1-\beta)q(\theta)} - \frac{p(1-x_i) + pc\theta + \nu}{r+\lambda+a(\theta)}, \tag{102}$$

$$S_i^{NS} = \frac{pc}{(1-\beta)q(\theta)} - \frac{p(1-x_i)}{r+\lambda}. \tag{103}$$